Name: ______________________________

Math 315, Section 2
Final Exam
Instructor: David G. Wright
17 April 2009, 11 AM – 2 PM

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1. (20%) Give an example of each of the following or argue that such a request is impossible:

   (a) a nested sequence of open intervals whose intersection is empty;

   (b) a bounded monotone sequence that has a divergent subsequence;

   (c) a sequence of integrable functions f_n defined on a closed interval $[a, b]$ that converges pointwise to a bounded function f that is not integrable;

   (d) a sequence of integrable functions f_n defined on a closed interval $[a, b]$ that converges pointwise to an integrable function f on $[a, b]$ so that $\lim_{n\to\infty} \int_a^b f_n \neq \int_a^b f$;

   (e) an infinitely differentiable function g so that $g(x)$ is equal to its Taylor series only if $x = 0$.

2. (10%) Show that if $0 < r < 1$, then $\lim_{n\to\infty} r^n = 0$.

3. (10%) Complete the following definitions:

   (a) A set C in $\mathbb{R}$ is *compact* if

   (b) Let $f : A \to \mathbb{R}$ be a function. Then f is *uniformly continuous* means

4. (10%) Prove that a continuous function on a compact set has a maximum; i.e., if $f : K \to \mathbb{R}$ is a continuous function and K is compact, then there is a $c \in K$ so that $f(x) \leq f(c)$ for all $x \in K$.

5. (10%) Let f and g be functions defined on an interval A that are differentiable at some point $c \in A$. Show that $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$.

6. (10%) State and prove the Mean Value Theorem.

7. (10%) Let (f_n) be a sequence of functions defined on $A \subseteq \mathbb{R}$ that converges uniformly on A to a function f. If each f_n is continuous at $c \in A$, show f is continuous at c.

8. (10%) Show that if a power series $\sum_{n=0}^{\infty} a_n x^n$ converges at some point $x_0 \in \mathbb{R}$, then it converges absolutely for any x satisfying $|x| < |x_0|$.

9. (10%) State and prove either version (not both) of the Fundamental Theorem of Calculus.